HEGEL, PEIRCE, AND ARISTOTLE ON THE "GEOMETRIC" LOGIC OF PRACTICAL REASON*

HEGEL, PEIRCE E ARISTÓTELES ACERCA DA LÓGICA "GEOMÉTRICA" DA RAZÃO PRÁTICA

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RESUMO: Este artigo examina uma convergência entre abordagens da razão prática nas lógicas de Aristóteles, Hegel e Peirce, em torno de uma forma de inferência não-demonstrativa que procede de um modo regressivo, das conclusões às premissas de uma inferência dedutiva. Na *Ética a Nicômaco*, Aristóteles descreveu um tipo de deliberação prática desta forma e a ligou a um tipo de inferência utilizada pelos geômetras em relação aos seus diagramas construídos. Peirce descreveria uma forma similar de inferência que chamou de "abdução", e paralelos entre as três formas de inferência de Peirce — dedução, indução e abdução — são encontrados no tratamento de Hegel das três *figuras* do silogismo de Aristóteles, no Livro III da *Ciência da Lógica*. Argumenta-se que esta postulação de uma terceira forma de inferência em Aristóteles é coerente com a reconstrução *platônica* de Hegel da silogística formal de Aristóteles e de sua relacionada separação das categorias da *singularidade* e da *particularidade*.

PALAVRAS-CHAVE: Hegel, Peirce, Aristóteles, abdução, lógica prática, lógica geométrica

ABSTRACT: This article examines a convergence between approaches to practical reason in the logics of Aristotle, Hegel and Peirce around a form of non-demonstrative inference that proceeds in a regressive way from conclusions to premises of a deductive inference. In *Nicomachean Ethics* Aristotle had described a type of practical deliberation in this way and had likened it to a type of inference used by geometers in relation to their constructed diagrams. Peirce would describe a similar form of inference he called "abduction", and parallels between Peirce's three inference forms—deduction, induction, and abduction—are found in Hegel's treatment of the three *figures* of Aristotle's syllogism in Book III of *The Science of Logic.* It is argued that this postulation of a third inference form in Aristotle coheres with Hegel's *Platonic* reconstruction of Aristotle's formal syllogistic and his related separation of the categories of *singularity* and *particularity*.

KEYWORDS: Hegel, Peirce, Aristotle, abduction, practical logic, geometric logic

1. Introduction

In his commentary on Euclid's *Elements*, Proclus, the last of the classical Platonist philosophers, described a type of inference used by Euclid as "geometrical conversion".[1]

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* Artigo convidado.

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[1] PROCLUS. **A Commentary on the First Book of Euclid's Elements**. Trans. G.R. Morrow. Princeton: Princeton University Press, 1970. Paterson discusses the strongly Proclean character of Hegel's attitude to geometry on the basis of notes from his study of Euclidean geometry in 1800. PATERSON, A.L.T. Hegel's Early Geometry, **Hegel-Studien**, vol. 39/40, 2005, pp. 61-124.

"Conversion" here refers to the "converse" direction taken by this inference which starts from the *conclusion* of a deductive inference and reasons to some hypothetical premise from which the conclusion could be deduced.

> If, for example, a theorem … arrives at a conclusion from several hypotheses, we take the conclusion and one hypothesis and reach a conclusion consisting of one or more of the other hypotheses (PROCLUS, 1970, pp. 196-197).[2]

So-called "conversion rules" also played a key role in Aristotle's logic as set out in the *Prior Analytics*, although there are important differences between Euclid's conversions and those found within Aristotle.[3] However, in his *Nicomachean Ethics*, and so outside the framework of the works comprising his logical "organon", Aristotle would appeal to his own version of a regressive type of inference from a conclusion of a deductive premise to the principles from which it can be deduced, likening it to a form of inference used by geometers in relation to their diagrams. Aristotle's own geometrical conversions were specifically relevant to *practical* reason, however.

It is generally thought that much of the geometry organized by Euclid into the thirteen books of the *Elements* had been developed by mathematicians associated with the Academy founded by Plato in 387 BCE, probably about a century before Euclid composed the *Elements* and eight centuries before Proclus would write his commentary. Aristotle had joined the Academy as a teenager about twenty years after its opening, and at a time when Eudoxus of Cnidus, a major contributor to Euclid's *Elements*, was active there.[4] It is often suggested that the approach of the geometry practiced in the academy,[5] and especially that of Eudoxus,[6] had provided an important model for Aristotle's syllogistic logic. Over two thousand years later, the American philosopher/logician Charles Sanders Peirce would argue for an hitherto unrecognized type of inference he believed he had found in Aristotle, that he initially called "hypothesis" and later "abduction". While not directly modelled on the practical inference form in *Nicomachean Ethics*, it was in many ways similar to that type of *geometric* inference, Peirce

[2] PROCLUS. **A Commentary,** pp. 196-197.
[3] On the nature of the geometrical conversions as found in Euclid, see Heath's comments to EUCLID. **The Thirteen Books of Euclid's Elements**. Trans. and ed. T.L Heath, three volumes. New York: Dover, 1956, vol. 1, prop. 6.
[4] Thomas Heath discusses Aristotle's familiarity with Eudoxus' discoveries in HEATH, T. **Mathematics in Aristotle.** Abingdon: Routledge, 2016, pp. 1 and 111.
[5] E.g., CORCORAN, J. Aristotle's Prior Analytics and Boole's Laws of Thought. **History and Philosophy of Logic,** vol. 24, n. 4, 2003, pp. 261-288.
[6] See in particular, LASSERRE, F. **The Birth of Mathematics in the Age of Plato.** London: Hutchinson, 1964, p. 97.

also arguing for a strongly *diagrammatic* conception of logic in general and abduction in particular.

In this paper I pursue *three*-way parallels between Peirce's abduction, Aristotle's non-standard "geometric" type of practical inference, and the type of syllogistic inference that Hegel, in the "Subjective Logic" of *The Science of Logic*, introduces as a syllogism generated from the most developed form of judgment he presents there, the *judgment of the concept*.[7] Foremost among the parallels will be the practical implications of the fundamentally *evaluative* nature of the judgments involved, implications most obvious in relation to the examples Hegel gives of this judgment type, "this action is good" and "this house is *bad*". All things being equal, one would surely avoid being on the receiving end of another's *bad* act and would naturally choose to live in a *good* rather than a bad house, these concepts applied in such contexts being instances of what Bernard Williams had called "action-guiding" ones.[8] On examination, the forms of inference extending from such judgments in Hegel, Peirce and the Aristotle of *Nicomachean Ethics*, all exemplify, I will argue, patterns of reasoning that distinguishes *their* logics from the more standard approaches to logic stretching from Aristotle as traditionally understood to Kant and most modern formal logic—approaches that Hegel had criticised as logics of "the mere understanding" (*Der blosse Verstand*). Comparison between the ways in which both Peirce and Hegel interpret the syllogistic figures of Aristotle's logic brings out the implicit geometric features of Hegel's approach. Moreover, these parallels help illuminate Hegel's logical project and idealist philosophy more generally. Not only is Peirce's abductive inference relevant to the logic of practical reason in Hegel, Hegel's entire logic might thereby be seen to concern judgments and inferences that are conceived primarily *as* actions carried out in the world.

2. From Aristotle's Nonstandard Geometrical Inference to Peirce's Abduction

In *Prior Analytics* book I, chapter 2, Aristotle describes rules of "conversion" (*antistrophe*) as one of the ways in which judgments can be transformed into other judgments

[7] HEGEL, G.W.F. **Science of Logic.** Ed. and trans. G. di Giovanni. Cambridge: Cambridge University Press, 2010, p. 582; 12.85. The reference following page number is to volume and page numbers of the Meiner edition, HEGEL, G.W.F. **Gesammelte Werke,** Hamburg: Felix Meiner, 1968–. Di Giovanni's translation has sometimes been modified.

[8] WILLIAMS, B. **Ethics and the Limits of Philosophy**. London: Fontana, 1985, ch. 8. Of course, considered in isolation, the predicates "good" and "bad" do not suggest the *kind of* attraction or avoidance behaviour being "directed". The meaning of "bad" in the sentence "this house is bad" relies on the *kind* of the subject in question.

by reversing subject and predicate terms. For example, "no As are B" can be converted into "no Bs are A". Aristotle's word "*antistrophe*" derived from the name of a particular dance made by the chorus in a tragedy. An *antistrophe* repeated an earlier dance, the *strophe*, but in the *antistrophe* the dance was performed from right to left rather than left to right. Aristotle's use of the term fits in with the fact that many of the technical terms common to both his logic and Greek geometry had originated with Pythagorean music theory with its concerns with ratios and the division of intervals.[9]

Conversions play an important role in Aristotle's syllogistic logical system, allowing some non-obvious syllogistic inferences to be translated into or "reduced" to "perfect" or "complete" (*teleios*) ones, whose validity is supposedly immediately apparent and certain. For example, it may not at first be clear whether or not the inference "No As are B; All As are C; therefore, some Cs are not Bs", is valid, but with a sequence of conversions this can be translated into the "perfect" syllogism traditionally known as Barbara, "All As are B; All Bs are C; therefore, all As are C".[10] The validity of this syllogism is thought to require no further proof as it is immediately perceivable by all, giving it a type of axiomatic status. Euclid's geometric conversions described by Proclus, however, were different to this. While geometric conversions involved a "certain interchange among the component parts" within a theorem, they did not function *within* an overarching syllogistic *deductive* framework but were themselves understood as complete inferences with multiple parts. Moreover, Euclid's geometric conversions, according to Proclus, were inferences that started from the *conclusion* of a deductive inference and that reasoned to some hypothetical premise from which it could be deduced. As in the dance, here the *antistrophe* repeated the sequence of the presupposed *stroph*, but in a direction that was contrary to, "*anti*", its predecessor.

Aristotle describes a type of inference in *Prior Analytics* that operates in a way that utilizes such a reversal of direction or rearrangement of order characteristic of conversion. This he calls "*epagoge*", an inference which is

> opposed to [*antiketai*] syllogism, for the latter shows by the middle term that the major extreme applies to the third, while the former shows by means of

[9] On the links of the nomenclature of Pythagorean music theory to Aristotle's syllogistic, see EINARSON, Benedict. On Certain Mathematical Terms in Aristotle's Logic: Parts I and II. **The American Journal of Philology**, vol. 57, n. 1 and n. 2, 1936, pp. 33-54 and pp. 151-72. On the to geometry more generally, see SZABÓ, A. **The Beginnings of Greek Mathematics**. Reidel: Springer, 1978.

[10] While this is a common way of representing this syllogism, it does not reflect the way Aristotle orders the terms. We will return to this issue below.

> the third that the major extreme applies to the middle (ARISTOTLE, 1938, 68b33-37).[11]

While syllogistic inference is from the general to the more particular, *epagoge* goes in the reverse direction. Aristotle's *epagoge* is traditionally translated as "induction", but it clearly differs from induction as understood within modern thought. While modern induction is considered of as generalizing from some array of immediately known entities to something holding universally of them, such as a law, Aristotle's *epagoge* seems to presuppose knowledge of the relevant general. For example, it has been described as a form of argument in which

> the learner comes to see the application of the general principle to a case as a result of constructing and using suitable cases (HAMLYN, 1976, p. 171).[12]

But Aristotle has frustratingly little to say about *epagoge*,[13] and what he does say is, according to many, unclear and equivocal.[14] However, we find a similarly inverted inference in *Nicomachean Ethics*, which Aristotle likens to a geometrical inference.

There Aristotle appeals to a form of reasoning as appropriate for practical "deliberation" that, in Book III, he compares to "the analysis of a figure in geometry" such that "the last step in the analysis seems to be the first step in the execution of the design".[15] Aristotle here uses "analysis" in a way that was specific to Greek geometry: as Proclus would note in his later history of geometry, in analysis the geometer passes "in the reverse direction" to that employed in demonstrations: rather than argue from "premises to conclusions", in analysis one passes "from conclusions to principles".[16] This fits the earlier description of practical deliberation that Aristotle had given in Book I where he had emphasized the difference in directionality between reasoning from first principles and deliberative reasoning *to* first principles, using the analogy

[11] ARISTOTLE, **Prior Analytics.** Tran. H. Tredennick. Cambridge, Mass.: Harvard University Press, 1938
[12] HAMLYN, D.W. Aristotelian Epagoge. **Phronesis: A Journal of Ancient Philosophy,** vol. 21, 1976, pp. 167-184.
[13] The accounts mostly referred to occur in ARISTOTLE, **Prior Analytics,** Bk. 2, ch. 21, ARISTOTLE. **Posterior Analytics**. Trans. H. Tredennick. Cambridge, Mass.: Harvard University Press, 1960, bk. 1, ch. 2.
[14] See the summary account of these debates in MCKIRAHAN, R.D. Aristotelian Epagoge in Prior Analytics 2. 21 and Posterior Analytics 1. 1. **Journal of the History of Philosophy,** vol. 21, n. 1, 1983, pp. 1-13, in which the author attempts to find a unifying account.
[15] ARISTOTLE, **Nicomachean Ethics.** Trans. H. Rackham, 2nd. ed. Cambridge, Mass.: Harvard University Press, 1934, 1112b20-25.
[16] PROCLUS. **A Commentary,** p. 57. (Proclus' commentary a Prologue containing a history of geometry from the Egyptians to his present.) Analysis in this sense was central to the so-called "problems" approach that was particularly important after Euclid. For a comprehensive account see KNORR, W.R. **The Ancient Tradition of Geometric Problems**. Boston: Birkhäuser, 1986.

of a race track along which a runner can run "from the judges to the far end or reversely".[17] Aristotle notes that while one must, as Plato had stated, "start from the known", this "known" can have two meanings, "what is known to us" and "what is knowable in itself". Axioms, providing the starting point of traditional Euclidean proofs, provided something "knowable in itself", and mathematical axioms had clearly provided Plato with a type of model for those self-evident "ideas" from which all reasoning was meant to flow. But, consistent with the generally more "empiricist" dimension of his own approach that he opposed to Plato's, Aristotle then adds that "perhaps then for us it is proper to start from what is known to us".[18] In this reversal of the direction of reasoning, "the starting point or principle [*archai*] is the fact that a thing is so [*to hoti*]", a worldly fact that can be reliably perceived by someone well-trained, and the inference takes the knower to the knowledge of "why it is so" [*ton dioti*].[19]

Almost two millennia after Aristotle, the American logician and pragmatist philosopher, Charles Sanders Peirce, would describe a similar type of inference that he would first call "hypothesis" and then later, "abduction", the name by which it is known today, adding it to the more conventional division of inferences into deductive and inductive ones.[20] Like Aristotle, Peirce would similarly treat induction as involving a reversal of the direction of a syllogistic inference. Peirce even employs a spatial metaphor not unlike Aristotle's *antistrophe* or that of running up or back along a racetrack, when he notes that one may "row … up the current of deductive sequence and [conclude] a rule from the observation of a result in a certain case". However, "this is not the only way of inverting a deductive syllogism so as to produce a synthetic inference".[21] Given that the syllogism has exactly *two* premises, he notes that besides arguing from result and case to rule as in induction, one might also argue from "result" and "rule" *to* "case"—that is, from conclusion and major premise to the minor—this is abduction.[22]

[17] ARISTOTLE. **Nicomachean Ethics**, 1095a34-b1. Aristotle seems to have in mind the type of track where one turns at the end of the course and returns to the finish where one started.
[18] ARISTOTLE. **Nicomachean Ethics**, 1095a29–b8.
[19] Aristotle discusses this difference between "knowledge of a fact and knowledge of the reason for it" and the type of inference that proceeds from the former to the latter in Aristotle, **Posterior Analytics**, bk. I, ch. 13. There, the example is given of inferring *from* the familiar fact that planets do not twinkle to the judgment that they are near. This must be an "ascending" type of inference rather than a syllogistic deduction from an effect to a cause. The nearness of the planets is the *reason* for their not twinkling. It is *not* the case that not twinkling is the reason of their nearness.
[20] See, for example: PEIRCE. C.S. "Some Consequences of Four Incapacities" and "Deduction, Induction and Hypothesis". In: **The Essential Peirce: Selected Philosophical Writings, Volume 1 (1867–1893)**. Eds. N. Houser and C Kloesel, Bloomington: Indiana University Press, 1992, pp. 28-55 and 186-199.
[21] PEIRCE. **The Essential Peirce, Volume 1,** p. 188.
[22] This all depends on treating syllogisms as having a fundamentally diagrammatic dimension: "Why do the logicians like to state a syllogism by writing the major premise on one line and the minor below it, with letters

Peirce would later claim that Aristotle himself had, in *Prior Analytics* Book II, chapter 25, distinguished abduction from induction when he described an inference type as *apogoge*, (standardly translated as "reduction") from *epagoge*, induction. Recently it has been argued that while Peirce's non-standard interpretation of this passage is less than convincing, other passages from Aristotle's *Posterior Analytics* do seem to conform to Peirce's conception of *abduction*.[23] Significantly, these include instances of the types of inferences from "that a thing is so" to "why it is so" alluded to above. Peirce would also conceive of reasoning in strongly "geometric" ways, claiming, for example, that all necessary reasoning is "of the nature of mathematical reasoning" and that "mathematic reasoning is diagrammatic".[24] Given the centrality of abduction in Peirce's logical scheme it is not surprising that one interpreter would refer to abduction itself as "meta-diagrammatic".[25] I suggest we view Peirce's distinction between these two different patterns of non-deductive inference as disambiguating Aristotle's equivocal *epagoge*, a disambiguation that in turn coheres with one we find in Hegel relating to the *terms* making up Aristotle's syllogisms.

Aristotle, presumably following the way in which Greek geometers had labelled their diagrams, labels his syllogisms with letters drawn from the Greek alphabet such as A, B, and Γ meant to stand as meaningless placeholders for terms playing the role of subject or predicate in the component sentences.[26] Hegel, however, would use the abbreviations E, B, A for *Einzelheit* (singularity), *Besonderheit* (particularity), and *Allgemeinheit* (universality).[27] This clearly fits with his characterization of the syllogism in the *Encyclopedia Logic*, where he describes the syllogism "in its truth" and in contrast to "the meaning it has in the old, formal logic", as

> that determination in virtue of which the particular is supposed to be the middle that joins the extremes of the universal and the singular together. This form of syllogistic inference is a universal form of all things. Everything is

substituted for the subject and predicates? It is merely because the reasoner has to notice that relation between the parts of those premises which such a diagram brings into prominence". PEIRCE, C.S. **Philosophy of Mathematics: Selected Writings**. Ed. M.E. Moore, Bloomington: Indiana University Press, 2010, p. 20. The idea of moving through syllogistic structures *via* different paths exemplifies what Peirce describes as "making experiments upon diagrams and the like and … observing the results" and which constitutes the "very life of mathematical thinking" (p. 40).

[23] FLÓREZ, J.A. Peirce's Theory of the Origin of Abduction in Aristotle. **Transactions of the Charles S. Peirce Society**, vol. 50, n. 2, 2014, pp. 265-280.

[24] PEIRCE, C. S. **The Essential Peirce: Selected Philosophical Writings, Volume 2 (1893–1913)**. Ed. the Peirce Edition Project. Bloomington: Indiana University Press, 1998, p. 206.

[25] HOFFMANN, M. 'Theoric Transformations' and a New Classification of Abductive Inferences. **Transactions of the Charles S. Peirce Society**, vol. 46, n. 4, 2010, pp. 570-590, p. 581.

[26] For simplicity we will henceforth use the English equivalents, A, B, C.

[27] From here on we will use the English equivalents, S, P, and U.

> something particular that joins itself as something universal with the singular (HEGEL, 2010, § 24, add. 2).[28]

As we will see, Aristotle has no official place *for* singular terms in his syllogisms and, from Hegel's perspective, blurs the distinction between the categories of singularity and particularity upon which he insists. We will explore this further in more detail below, but first it is important to get an initial understanding of the parallels between Peirce and Hegel in relation to Peirce's posited third inferential form, abduction.

3. Peircean and Hegelian Inferences

In his comparatively neglected "subjective logic" in Book III of *The Science of Logic* Hegel attempts to sketch out the logical nature of concepts, judgments and syllogisms in a genetic way that proceeds by a series of self-corrections from immediate to progressively "mediated" forms. Rejecting the axiomatic approach, Hegel shows those initial forms, that might *seem* to be appropriately self-evident starting points for reasoning, are in fact riven by contradictions and in need of redetermination. Thus, starting from concepts that seem to be individually meaningful, these concepts are shown to be only properly understood when considered as components of judgments. In turn, judgments are shown to be meaningful only when functioning as components within syllogisms.

When this dialectic is traced out within syllogisms, initially *abstract* forms like Aristotle's self-evident perfect syllogisms come to be redetermined in more and more concrete ways, leading to a metaphysical conception of the whole of concrete existence as syllogistically structured. As an *objectivity* in this way, Hegel's ultimate syllogism seems to have its origins in Plato's account of the rational structure of the living cosmos in the dialogue *Timaeus*,[29] from which he describes Aristotle as had derived his own *formal* syllogism which is, however, restricted to a logic of the understanding.[30] In his account of the syllogism in *The Science of Logic* Hegel attempts to show how Aristotle's abstract formal syllogism—the syllogisms most immediate form—transitions, under the pressure of its own internal contradictions, into the

[28] HEGEL, G. W. F. **Encyclopedia of the Philosophical Sciences in Basic Outline, Part I: Science of Logic**. Trans. and ed. K. Brinkmann and D.O. Dahlstrom, Cambridge: Cambridge University Press, 2010; c.f., § 181, add.
[29] PLATO, Timaeus. In: **Complete Works.** Ed. J.M. Cooper, J.M, Indianapolis: Hackett, 1997, 29e-41d.
[30] HEGEL, G. W, F. **Lectures on the History of Philosophy, *1825–6. Volume II: Greek Philosophy***. Trans. and ed. R. F., Oxford: Clarendon Press, 2006, pp. 207-215. Hegel's discussion on page 210 of the derivative nature of the "syllogism of the understanding" is a clear reference to Aristotle's syllogism as conventionally understood.

properly *rational* and *concrete* syllogism, like that of Plato. But in order to understand this properly we must grasp how *Hegel's* Plato was not Plato as popularly understood as the proponent of some transcendent beyond, populated by immediately cognizable immaterial "ideas". Rather, influenced by later neo-Platonists like Proclus, Hegel took Plato to have treated these ideas as necessarily *self-actualizing*, resulting in the type of corporealization of essences appearing in space and time that Aristotle had taken as the starting point of his opposed "empiricist" and "inductive" approach. Moreover, Hegel would insist on the distinction between concrete "singulars", *Einzelnen*, and the abstractly conceptual "particulars", *Besonderen,* that those singulars actualize, to reflect this neo-Platonic heritage. This is why Aristotle's tendency to blur these categories needed to be corrected and the determinations of *singularity*, *particularity*, and *universality* restored to their proper "dialectical" unity in logic.

Hegel's complex account of the evolving shapes of judgment in Book III of *The Science of Logic* proceeds through a number of cycles, starting with the type of simple perceptual judgments that modern empiricists might take as the starting point of inductive reasoning. Such a notion of what a judgment is, however, will be grasped as self-contradictory and will generate another, more "mediated" conception of judgment meant to correct its deficiencies. Thus, early on, Hegel distinguishes broadly between two judgment forms that treat predication in different ways. In the first, predicates are taken as "inhering" in their subjects, reflecting in the way that an immediate property of a substance, a tomato's redness for example, is perceived as inhering in it.[31] This concept of a judgment will be shown to contain contradictions, however, and will transform into a more complex one in which the predicate of a judgment is now understood as "subsuming" the subject.[32] We might think of this latter form as one in which the predicate is taken as expressing a concept which is *true* of that subject. Proclus had a suggestively similar distinction, when, as part of a three-fold distinction among "universal forms", he distinguished "the universal shared in by its particulars" and "the universal in its particulars".[33]

Broadly, Hegel's distinction between these two basic judgment forms aligns with what scholastic logicians had discussed as *de re* and *de dicto* interpretations of judgments. On the former interpretation, the judgment's content or meaning is thought to unproblematically

[31] On the nature of judgments of inherence, see HEGEL, **Science of Logic,** p. 555: 12.57-58. For examples, see pp. 558-559; 12.61-62.
[32] This judgment is contrasted with judgments of inherence. See HEGEL, **Science of Logic,** pp. 555; 12.58 and 570; 12.72, while examples are found at pp. 568-569; 12.71-72.
[33] PROCLUS. **A Commentary,** p. 41. The third type of universal distinguished in this passage is one that "supplements the particulars".

contain the thing (*res*) it is about, on the latter, the content or meaning is taken to be what is determinately *thought* or *said* (*dictum*) concerning the thing. Significantly, Aristotle is often taken to have confounded this distinction.[34] While for Hegel judgments of subsumption "negate" the earlier judgments of inherence, they too will be subject to the same logical breakdown and redetermination and will be replaced by a new form of judgment in which the structures of *both* earlier forms have been, in Hegel's somewhat trademark way, "sublated" [*aufgehoben*]—that is, both negated and yet retained. In this case, such sublation will result in a *new* type of judgment that incorporates aspects of both earlier sublated forms, and this new form will set off a subsequent cycle. Judgments instantiating this new form, the "judgment of necessity",[35] will effectively have as their subjects Aristotelian "kinds" or "secondary substances". Whereas judgments of the first cycle may have been about individual roses, those of the second cycle might be about "the genus" rose, for example. This new judgment form passes through a similar cycle of logical breakdown and redetermination, ultimately issuing in a new and final type of judgment, the "judgment of the concept". This is the most developed conception of judgment and it will transition into a syllogism.

From Hegel's examples it appears that what distinguishes the subjects of judgments of the concept from those of earlier judgment forms is that they are either *human actions* or *products* of those actions, artifacts, and that what is predicated of them is either one of two predicates: they are judged to be *good* or *bad.* These evaluative judgments such as "this house is bad" or "this act is good" will expand such that a particular term, such as "as so and so constituted" will be inserted *between* a singular subject and its universal predicate, resulting in a complex "apodictic" judgment as in "the house, as so and so constituted, is *good*". A complex judgment of this type can now be restructured as a syllogism, "this house is so and so constituted; houses so and so constituted are good; therefore, this house is good",[36] in which the minor premise states the properties accounting for that goodness or badness of the house, thereby giving *reasons* for the judgment. What had been immediately perceived as an Aristotelian "that it is so" has led to a judgment of "the reason why it is so".[37] The need for this expansion, we are led to believe, is because of the highly *subjective* nature of the initial evaluation. The immediate "assertoric" form of the judgment of the concept had for its

[34] E.g., STRIKER, G. Introduction and Commentary. In: Aristotle, **Prior Analytics Book 1**. Trans. G. Striker, Oxford: Clarendon Press, 2009, p. 111.
[35] HEGEL. **Science of Logic,** pp. 575-576; 12.77-78.
[36] HEGEL. **Science of Logic,** p. 585; 12.87.
[37] ARISTOTLE. **Nicomachean Ethics,** 1095b7-9.

"credential" only a "subjective assurance" and this contingency will lead to its being "confronted by an opposing one" which, as equally contingent, has "equal justification".[38]

The giving of reasons that is the transition to the syllogism is clearly a response to such opposition, such evaluations of human acts or artifacts being applications of what have been called "essential contested concepts".[39] But *acts* of judging are, of course, themselves actions that are also typically contested. This means that the judgment of the concept has also now allowed concrete *acts of judgment* to be located within that same logical space as the acts or artefacts being judged. Each of the opposing judgments made by the interlocutors is a possibly good or possibly bad instance of what a judgment in its essence is—that is, a possibly good or bad instantiation of the very *concept* of what it is to be a judgment. This is why for Hegel, with the judgment of the concept, the *concept of judgment* itself has reached its completed or self-sufficient form. It is this *completed* structure of judgment transitioning into syllogism that I want to liken to Peirce's perceptual judgments that feed into abduction *via* analogies they both have to Kant's judgment of aesthetic taste.

The subject of Hegel's judgment of the concept is resolutely *singular*, Hegel employing here demonstrative phrases "this house", "this action", and the significance of its singularity as *opposed to* particularity can be brought out by appealing to some formal similarities that Hegel's judgment bears to Kant's treatment of judgment of aesthetic taste, those "judgments of reflection" as treated in the first part of his *Critique of the Power of Judgment*, despite some obvious differences. [40] "In regard to logical quantity all judgments of taste are", states Kant, "singular judgments".[41] This is crucial because judgments of taste can *only* be made in the direct presence of the object judged. "I must immediately hold the object up to my feeling of

[38] HEGEL. **Science of Logic,** pp. 583-584; 12.85-86.
[39] The conception of "essentially contested concepts" that are applied within human activities was developed by W. B. Gallie who lists a series of conditions for a concept to essentially contested. It "must be *appraisive* in the sense that it signifies or accredits some kind of valued achievement". The achievements to which it applies must have an "internally complex character", such that "any explanation of its worth must … include reference to the respective contributions of its various parts of features". It "must be of a kind that admits to considerable modification in the light of changing circumstances" and each party to a contestation must "recognises the fact that its own use of it is contested by those other parties, and that each party must have at least some appreciation of the different criteria in the light of which the other parties claim to be applying the concept in question". GALLIE, W.B. Essentially Contested Concepts. **Philosophy and the Historical Understanding.** London: Chatto and Windus, 1964, p. 161. The predicates of Hegel's judgments of the concept, I suggest, effectively meet all these criteria.
[40] They also have features associated with the judgments of reflection treated in part II of the *Critique of the Power of Judgment*, judgments about *organisms*, but for our purposes I will restrict the parallel to judgments of taste.
[41] KANT, I. **Critique of the Power of Judgment.** Ed. P. Guyer, trans. P. Guyer and E. Matthews, Cambridge: Cambridge University Press, 2000, § 8.

pleasure and displeasure". One cannot learn that some object is beautiful *on the basis of* an inference from the knowledge that it has properties conforming to some general *principles* of beauty. There can be no principles of taste

> under the condition of which one could subsume the concept of an object and then by means of an inference conclude that it is beautiful … for I must be sensitive of the pleasure immediately in the representation in it, and I cannot be talked into it by means of any proofs (KANT, 2000, § 34).

Kant's judgment is an Aristotelian judgment that something is so (*to hoti*), not *why* it is so (*ton dioti*).

Another feature common to these two judgment forms is the explicit contrariness of their possible predicates. For Kant, the opposite of beauty is ugliness, not simple *lack of* beauty. Similarly for Hegel, the possible initial reactions to the house or actions seem to come in contrary pairs, with the opposed positive and negative characteristics of subjective responses to such objects. Hegel's examples indicate that the goodness or badness involved in judgments of the concept are not specifically *aesthetic* goodness or badness—not beauty or ugliness—while Kant's judgments of taste do not *extend beyond* aesthetic goodness to, say, *good* acts or artifacts such as houses. However, these differences are bound up with their different interpretations to the logical category of singularity itself. For Kant, concepts are *necessarily* general and so singularity becomes the quantity of a *non*-conceptual form of cognition that he calls *intuition*.[42] In contrast and as we will expect from his characterization of the syllogism, Hegel includes singularity within the structure of conceptuality itself: singularity being, along with particularity and universality, a "moment" of "the concept".[43] It is this that gives singular subjects in Hegel an important *official* role in properly syllogistic inference, in contrast to the *unofficial* role played in Aristotle, and the *lack of* any proper role for Kant.

For his part, Peirce treats perceptual judgments, like Hegel's immediate judgments of the concept, as smoothly transitioning into a type of syllogism. Direct perceptual judgment is a judgment into which "abductive inference shades … without any sharp line of demarcation

[42] "A perception that refers to the subject as a modification of its state is a sensation (*sensatio*); an objective perception is a cognition (*cognitio*). The latter is either an intuition or a concept (*intuitus vel conceptus*). The former is immediately related to the object and is singular; the latter is mediate, by means of a mark, which can be common to several things." KANT, I. **Critique of Pure Reason**. Eds. P. Guyer and A.W. Wood, Cambridge: Cambridge University Press, 1998, A320-321/B376-7.
[43] HEGEL. **Science of Logic,** pp. 546-549; 12.49-52.

between them".[44] While Peirce is not particularly clear or consistent in his account of perception,[45] we might say that for him the type of judgment that is the basis for this peculiar type of inference is broadly characterized in a Kantian "aesthetic" way. In a lecture he treats the normativity of *logic* as dependent on that of *ethics* which is in turn dependent on that found in "*esthetics*".[46] An act of inference "consists in the thought that the inferred conclusion is true because *in any analogous case,* an analogous conclusion *would be true*" and to describe an inference is logical is an *approval* of it and in this way "the logically good is simply a particular species of the morally good" which, he goes on to argue "appears as a particular species of the esthetically good". More particularly, the type of perceptual judgment he describes as the basis of abduction typically has a creative and interpretive character, involving a spontaneous grasp of the unity or orderliness inherent in the perceptive experience in a way that is suggestively aesthetic.[47]

Again, we find something analogous within Aristotle's geometrically conceived form of practical judgment discussed in the *Nicomachean Ethics*. Just as Hegel's judgment of the concept makes it explicit that the act of judging is something that can be done well or badly, in Aristotle's account of practical reason about "the Right and the Just, and in … the topics of Politics in general", it becomes important that the judge (given the context, Aristotle says the "pupil") "is bound to have been well trained in his habits".[48] Being grounded in a type of mechanically reflex response to some experienceable state of affairs does not exclude the judgment from being a "logical" judgment, as it seems to in Kant. A *well-trained* judge is likely to judge *well*, a poorly trained one less likely. Judges, it would seem, are fallible but capable of learning. What might the nature of this learning process be? We might once more be guided by Peirce's approach.

Let's say the judgments of a pupil are corrected by those of a better-trained teacher who from a well-judged "mere fact" infers to its principle. In relation to Peirce's "meta-

[44] PEIRCE. **The Essential Peirce, Volume 2,** p. 227. Or alternatively, such perceptual judgments "are to be regarded as an extreme case of abductive inferences".
[45] See, for example, ROSENTHAL, S Peirce's Pragmatic Account of Perception. In: MISAK, C. (ed.), **The Cambridge Companion to Peirce**. Cambridge: Cambridge University Press, 2004, pp. 193-213.
[46] PEIRCE. **The Essential Peirce, Volume 2,** pp. 200-201. This is a characteristically Platonic unity of the good, the true and the beautiful.
[47] In "Pragmatism as the Logic of Abduction" Peirce describes as the second of the three "cotary" propositions of pragmatism that "perceptual judgments contain general elements" or concepts, abduction being effectively the route of the interpretative expansion of what is given directly in the experience. PEIRCE. **The Essential Peirce, Volume 2,** pp. 227-229.
[48] ARISTOTLE. **Nicomachean Ethics,** 1095b5-7.

diagrammatic abduction" Hoffmann points out an instance where Peirce had invoked an analogy from geometry, but not from Euclidean geometry, rather from the discipline of projective geometry that had blossomed in the nineteenth century but that had roots in the post-Euclidean phase of Greek geometry. Projective geometry, as the name suggests, is not so much concerned with the properties of plane figures—circles, triangles, rectangles, and so on—as found in Euclidean geometry. Rather, it considers relations among *projections* of three-dimensional objects onto differently oriented two-dimensional surfaces. A coin might be projected onto one plane as a circle but onto a differently oriented plane as an ellipse, just as it can *look* circular from one angle but elliptical from another. Peirce seems to have considered this as a model for alternate ways of *choosing* concepts when *describing* an object. In Hoffmann's words, "our vantage point determines the set of available theoretical models. It is possible to generate new models simply by shifting the perspective on a problem".[49]

Here we might presumably take the idea of "viewpoint" or "perspective" as itself an extension of the underlying literal meaning of the term to include the particular concepts we bring to it in attempting to describe it. What Aristotle's pupil might learn from the teacher is thus a *new way of seeing* the same object by appreciating how it *could* be described in a different way, as in judging, one must choose from which of a variety of possible descriptive concepts to bring to the object. With this in mind, we can appreciate how in Hegel's case, the judgment of the concept should be thought of as leading into an inference to the *minor premise* of a syllogism, the premise that specifies *the particular* concept (from the *many* that are true of it) that is relevant to its goodness or badness.

4. Aristotle's Syllogistic Figures

Since the development of modern logic from the end of the nineteenth century, few are likely to be familiar with the details of Aristotle's syllogistic logic. If the word "syllogism" conjures up anything, it will probably be something of the form "All As are B; All Bs are C; therefore, all As are C", or perhaps the familiar example about Socrates, "All humans are mortal, Socrates is human, therefore Socrates is mortal". However, neither of these purported syllogisms strictly fit Aristotle's descriptions of a syllogism, but for different reasons.

[49] HOFFMANN, Theoric, p. 581. In "Pragmatism as the Logic of Abduction" Peirce describes a similar possible three-dimensional interpretation of a two-dimensional drawing, such as a figure of a pair of steps that can be understood as being viewed from above or below. PEIRCE. **The Essential Peirce, Volume 2,** p. 228.

For its part, the status of the "Socrates syllogism" *as a syllogism* is questionable.[50] In the "old formal logic" to which Hegel refers—effectively the traditional way in which Aristotle's logic had come to be understood—*singular terms* such as proper names typically had not appeared as the subjects of judgments.[51] In line with this, in the first paragraphs of *Prior Analytics* Aristotle describes a premise as "a sentence that affirms or denies something of something, and this is either universal or particular or indeterminate",[52] noticeably omitting *singular* judgments, that is, judgments with singular terms.[53] Nevertheless, such syllogisms are not totally without basis in Aristotle. As Patzig points out,[54] while Aristotle was "obviously inclined to exclude them", he had given a number of examples of syllogisms using singular terms. Aristotle's ambivalence here points to the fact that for him the distinction between "singular" and "particular" which Hegel emphasises was at best implicit. While favouring a strictly "categorical" interpretation of the judgments involved in syllogisms, there are nevertheless suggestions of proper names having a role.[55] In exploring Aristotle's syllogism we will adopt the strict categorical reading to try to bring the surface what is at stake when Aristotle is interpreted consistently in this way.

As for the familiar "All As are B; All Bs are C; therefore, all As are C", Aristotle tended *not* to order the component sentences in the standard subject-predicate way that this suggests. To try to capture why syllogisms were valid, Aristotle invoked the idea of the transitivity of *containment relations* appropriate for either of the two perfect syllogisms that occur in the first figure and to which all syllogisms in the other two figures could be reduced:

> When three terms are so related to one another that the last is wholly contained in the middle and the middle is wholly contained in or excluded from the first,

[50] ŁUKASIEWICZ, J. **Aristotle's Syllogistic from the Standpoint of Modern Formal Logic.** Oxford: Oxford University Press, 1957, p. 1.
[51] The widespread use of singular terms did not appear until the Medieval nominalist logicians transformed the syllogism by treating singular terms as universals.
[52] ARISTOTLE. **Prior Analytics,** 24a16-18.
[53] The problematic status of singular terms would trouble later medieval *nominalist* logicians who would employ proper names but interpret them *as* universals, the justification being that with the assertion "Socrates is mortal", mortality is being attributed to *all*, not *part of*, Socrates. Leibniz would complicate matters further by also treating singulars *as* particulars, as Socrates might also be referred to as "some philosopher", the "some" used in the quantity of particularity not *excluding* just one. These strategies have been recently pursued in attempts to rehabilitate traditional logic. See, for example, SOMMERS, F. **The Logic of Natural Language.** Oxford: Oxford University Press, 1982 and ENGLEBRETSEN, G. Singular Terms and the Syllogistic. **The New Scholasticism**, vol. 54, 1980, pp. 68-74.
[54] PATZIG, G. **Aristotle's Theory of the Syllogism: A logico-philosophical study of Book A of the Prior Analytics.** Trans. J. Barnes, Dordrecht: Reidel, 1968, pp. 4-5.
[55] For a clear account of the reasons given by a variety of interpreters for the exclusion of singular terms in Aristotle's syllogisms, as well as arguments for their inclusion, see ENGLEBRETSEN, G. Singular, pp. 68-74.

> the extremes must admit of perfect syllogism (ARISTOTLE, 1938, 25b32-37).

That Aristotle conceives of the less general term as contained in the more general *container* is made clear when he immediately paraphrases this expression using the relation "is predicated of": "For if A is predicated of all B, and B of all C, A must necessarily pre predicated of all C".[56] In short, Aristotle is thinking of the constitutive sentences of this first figure syllogism as having a predicate-subject order, rather than the subject-predicate order that is in fact normal for both English and ancient Greek.[57]

The fundamental role given to the "is in" relation of containment here might suggest that Aristotle thinks of the basis of valid inferences as a type of application of the principle that if I know that there are chocolates (the last extreme) in a box (the middle), and I know that that box is in my bag (the first extreme), then I know that the chocolates are in my bag. A "middle" that is contained in one extreme and is contained in the other, here the box, can simply be dropped and the overall relation maintained between extremes. However, interpreting such containment "extensionally" in this way so as to conceive of the *individual members* of a species being contained in a general class that itself can be contained within the larger class would seem to presuppose a role for singular terms as the proper names for the specification of those individuals. In contrast, the strict categorical reading rules this out with the relevant "containment" relations now being thought to exist between *the classes themselves*—that is, abstract entities capable of both containing and being contained within *other classes*, as is required of the middle term of the perfect syllogism.[58] On this reading, we might think of this situation as pictured by a diagram in which an *area*, representing a genus, say animals, is partitioned into two subareas—those "parts" of the genus—representing subclasses of rational and non-rational animals, respectively, and which are capable of further partition. Adding weight to this interpretation is the fact that Eudoxus of Cnidus, whose theory of proportions seems to have deeply influenced Aristotle's syllogistic,[59] had conceived of the ratios and proportions between *continuous* magnitudes, such as lines or areas, to have determinate values

[56] ARISTOTLE. **Prior Analytics,** 25b38-40.
[57] See ROSE, L.E. Premise Order in Aristotle's Syllogistic. **Phronesis**, v. 11, n. 2, 1966, pp. 154-158.
[58] This is the situation in modern set theory, where strictly the only thing contained by sets are *other sets*. There are no distinct atomistic "members" as such. A set is not like a football team which has members that are not themselves *teams*.
[59] LASSERRE, **The Birth of Mathematics in the Age of Plato,** p. 97.

independently of being specified by pairs of numbers.[60] Numbers are the sorts of discrete quantities used to count individuals, and the apparent elimination of numbers in Eudoxus' geometry might have encouraged the elimination of singular terms in Aristotle's logic.

In his formal classification, Aristotle first divides syllogisms into first, second and third syllogistic "figures" (*schemata*) by the position of the "middle term" in each.[61] Clearly the two perfect syllogisms contained in the first figure were considered as the paradigm of a syllogism, and concordantly, the middle term (B), understood as the term that contains the last (C) and is contained by the first (A), is represented as standing *between them*, as in ABC. The inferential sequence here is between the two premises, AB and BC, to the conclusion, AC, in which the middle term has been dropped. This term, B, is here middle in two senses. It is what we might call the "containment middle" because it contains one term and is contained by the other, and it is middle in location, what we might call the "locational middle". The first figure might thus be thought of as perfect for the additional consideration that here containment and locational middles coincide, but in the two other figures they come apart. Thus, Aristotle writes that in the second figure, "the middle is placed outside the extremes and is first in position"[62] while in the third, "the middle is placed outside the extremes, and is last by position".[63] We can thus list the three syllogistic figures, ABC, BAC, and ACB, as having the following structures:

first figure, AB, BC, therefore AC;
second figure, BA, AC, therefore BC;
third figure, AC, CB, therefore AB.

Aristotle had seemingly used this model in a mechanical way to generate as many types of deductive argument as possible. Within each figure, different "moods" show different "ways" or "manners" in which the figures might be realized when the different quantities of "all" and "some" are assigned to the subject terms of the sentences together with the opposing "qualities"

[60] This was a crucial response to the problem of the "incommensurability" discovered between discrete and continuous magnitudes that had disrupted the Pythagorean belief that all ratios between continuous magnitudes could be reduced to ratios between discrete magnitudes. Aristotle thus thought of a line as capable of infinite, in the sense of unlimited, divisions. One did not eventually reach an array of atomic *points*, as had been conceived earlier by the Pythagoreans. It has been argued that Eudoxus had effectively *eliminated* numbers from mathematics. GARDIES,J.-L. **L'heritage epistemologique d'Eudoxe de Cnide: Un essai de reconstitution.** Paris: Vrin, 1988.
[61] "Schema" was the word used by Greek geometers for the diagrams accompanying proofs.
[62] ARISTOTLE. **Prior Analytics,** 26b37-38.
[63] ARISTOTLE. **Prior Analytics,** 28a14-15.

of affirmation or negation. Purely arithmetically, each figure could be realized in 64 ways but most of these are ruled out as being valid inferences, and hence syllogisms, by counterexamples. Across the three figures only 14 of these combinations turn out to be syllogisms. Both Peirce and Hegel would attempt to use Aristotle's classification into figures and moods in a more principled way, however, the three *figures* ultimately coinciding with the three functionally different forms of inference: deduction, induction, and abduction. This is most easily seen in the case of Peirce.

5. Functional Readings of the Syllogistic Figures by Peirce and Hegel

In an early paper from 1878 in which he refers to what he would later call "abduction" as "hypothesis", Peirce links the three syllogistic figures to the three forms of inferential reasoning. Here we find Peirce starting by being critical of Aristotle's would-be "perfect" first-figure syllogisms. Acknowledging that second- and third-figure syllogisms can be converted to the first, he states that "it does not follow that this is the most appropriate form in which to represent every kind of inference". In fact, the perfect syllogism is "nothing but the application of a rule" that, being laid down in the major premise, can be applied to a case stated in the minor to produce a result. Inductive and hypothetical (abductive) reasoning, however, both

> being something more than the mere application of a general rule to a particular case, can never be reduced to this form (PEIRCE, 1992, p. 187).

One might expect that the re-orderings of the three judgments in the syllogistic figures achieved by Aristotle by the movement of the middle term will coincide with that achieved by "rowing up" the deductive stream in either of the two alternate ways.

Peirce first gives an example of generating inductive and abductive inferences from a deductive one *via* an array of possible inferences that can be made when sampling populations, in the example, sampling beans drawn from different bags. He then turns to another way of generating these alternative non-deductive forms of inference from a deductive one that uses denial. Importantly, possibly employing the medieval strategy of allowing singular terms by treating them *as* universals, Peirce employs *singular* terms (the pair of names, Enoch and Elijah) in subject position in these syllogisms.[64] Again, starting with a syllogism in the first figure

[64] Remember that the relation among subject and predicate terms in Aristotle's second figure allows singular terms to be used as they are subjects in the two sentences in which they appear.

others can be derived. Together with the denial of the conclusion, one can deny either of the major or minor premises.

In this way, from Peirce's initial deductive syllogism "All men are mortal. Enoch and Elijah were men. Therefore, Enoch and Elijah were mortal", two further valid deductive inferences can be formed involving a denial of the result. First, if the result is denied while the rule is affirmed, we must infer the denial of the case: "Enoch and Elijah were not mortal. All men are mortal. Therefore, Enoch and Elijah were not men". On the other hand, if, having denied the result, we affirm the case, we must infer the denial of the rule: "Enoch and Elijah were not mortal. Enoch and Elijah were men. Therefore, some men are not mortal".[65] The inference in which denial of the result infers to the denial of the case turns out to be a mood of the second figure,[66] while that in which the denial of the result infers to the denial of the rule is in the third figure.[67] But also, the former, as an inference to the minor premise or "case", is a negative form of abduction, while the latter as an inference to the rule is a negative form of induction. Peirce has thereby repeated the alignment of his three forms of inference deduction, abduction and induction with Aristotle's three syllogistic figures that had been achieved with the earlier example involving sampling.

Striking parallels to Peirce's functional interpretation of Aristotle's syllogistic figures can be found in Hegel's own interpretation of Aristotle's formal syllogistic. As we have seen, Hegel introduces the syllogism as an expansion of a value judgment and operates in a way that has features like Peirce's abduction. After being introduced, however, the syllogism goes through a developmental cycle similar to that traversed earlier by judgment. Just as the most immediate judgment type had been the judgment of inherence labelled the "judgment of existence", the first syllogistic type is the "qualitative" syllogism of existence that transitions into the *quantitative* syllogism of reflection which in its turn transitions into the syllogism of necessity. Each of these syllogisms is in turn analysed has having three syllogistic components: those of the syllogism of existence are the traditional three *figures*.

While Aristotle's letters 'A,' 'B,' and 'C' had been used as meaningless placeholders, Hegel uses capitals 'S,' 'P,' and 'U' are themselves meaningful, signifying the three semantically related conceptual determinacies of singularity, particularity and universality.[68]

[65] PEIRCE. **The Essential Peirce,** p. 190.
[66] In the traditional classification of syllogisms, this is in the mood of "Baroco".
[67] This is in the mood of "Bocardo".
[68] Or, more properly, 'E,' 'B,' and 'A' correlating with *Einzelheit, Besonderheit,* and *Allgemeinheit.*

His order also reflects the more natural (to both German and Greek) word order of subject followed by predicate, meaning that his "SPU" for the first syllogism has the reverse word order to Aristotle's "ABC". Hegel's reflects the movement from subject to predicate and so from less to more general, while Aristotle's reflects the order from the more to less. This pattern is essentially repeated when the formal syllogism of existence, *via* the mathematical syllogism, transitions into the syllogism of reflection, with its substructures of syllogisms of "allness", "induction" and "analogy", and then the syllogism of necessity, with its substructures of categorical, hypothetical, and disjunctive syllogisms. As elsewhere, we observe the pattern of cycles being iterated within cycles.

Given the complexities introduced by Hegel's treatment of the syllogism via the peculiar categorical pattern within which his analysis unfolds, it might be thought ambitious to recover much in common between Hegel's treatment and the approaches of Aristotle and Peirce. Nevertheless, enough commonality can be observed to suggest a convergence between Peirce and Hegel here based on Hegel's explicit disambiguation of Aristotle's treatment of singularity and particularity. In particular, it can be appreciated that Hegel's three syllogisms internal to the syllogism of reflection, the syllogisms of allness, induction and analogy, correspond closely to Peirce's functional rendering of Aristotle's three figures as deductive, inductive and abductive. It is the role of order and directionality implicit in Aristotle's original geometric framing of his syllogistic, I suggest, that enables this. Hegel will use the idea of moving Aristotle's "middle term", with its dual senses of "middle", in either of the two directions defined by the "extremes" to capture something like the rearrangements that Peirce envisages as "rowing up" the deductive stream along two different routes, one which takes reasons from the conclusion and minor premiss to the major (induction) and one that takes reasons from the conclusion and the major premise to the minor (Peirce's abduction, Hegel's inference by analogy). A number of factors need to be ignored such as the complication in the *particular* ordering of these syllogistic figures resulting from Aristotle, for his part, and Hegel and Peirce, for theirs, employing reversed word orders within judgments.

Hegel's *second* subsyllogism of the reflective syllogism, the "syllogism of induction", reflecting the "PSU" ordering of his second figure, has "singularity for its middle term", however, "not *abstract* singularity but singularity as *completed*, that is to say, posited with its opposite determination, that of universality", reflecting the medieval nominalists' identification of singularity and universality. Hegel goes on:

> The one extreme is some predicate or other which is common to all these singulars; its connection with them makes up the kind of immediate premises, of which one was supposed to be the conclusion in the preceding syllogism (HEGEL, 2010, p. 612; 12.113).

This is clearly Peirce's account of induction treated as reversal of the immediately preceding first-figure deduction.

Consider, for example, the deductive syllogism "All foxglove plants are poisonous; all the plants in my garden are foxglove; therefore, this plant from my garden is poisonous". One could imagine testing the major premise by taking samples, *this* plant, *that* one*, that* one over there, and so on, and feeding them to animals in experiments. One might conclude that in fact, all foxglove plants, consumed in certain amounts, are poisonous. But, as in accounts of "hypothetico-deductive" inquiry, this needs some way of generating the hypotheses *to be* tested. Clearly Peirce's abduction was meant to provide this starting point. Hegel turns to "inference by analogy" to perform this task.

> The truth of the syllogism of induction is therefore a syllogism that has for its middle term a singularity which is immediately in itself universality. This is the syllogism of analogy (HEGEL, 2010, 614; 12.115).

There is clearly *something* of Peirce's abductively arrived at hypothesis in Hegel's syllogism of analogy. As in Hegel's example arguing *from* "*The earth* has inhabitants" and "the moon is *an earth*" (that is, is a thing of basically the same kind as *the* earth") to the conclusion "Therefore the moon has inhabitants" is to hypothesize. Unlike the situation in induction which relied on the nominalist treatment of a singular term as a universal to bring out its logical properties, this one relies on the alternative introduced by Leibniz, in which the substitution between quantities is between singular and *particular*. An analogy between the moon and the earth converts "the earth" from a proper to a common name. The moon is now considered as, like the earth, "an earth"—an instance of the type *planet*, revolving around a bigger body, its "sun".

Clearly this is a fairly crude type of hypothetical reasoning, although we might imagine something *like* this as a part of the "logic of discovery" phase of inquiry, as when the atom was conceived on the model of *our* solar system, and it might be thought as in need of the reciprocal "logic of verification," served by the process of *induction* as treated as part of the hypothetico-deductive process. Contrary to the common view where "conjectures" have no logical connection to the world in contrast to the falsifying "refutations" which they face, Peirce and

Hegel extend their conceptions of logic to that first phase of inquiry. This is the necessary role played by abduction/hypothesis/inference by analogy in the very *life* of thought, distributed over an actual community of minded individuals—ourselves.

6. Conclusion

I have stressed the practical contents and consequences of Hegel's judgment of the concept, a practical dimension that in a dialogical context of contested judgments can be extended metalogically. That *only* explicitly evaluative judgments of actions or their products can be extended to encompass the very act of judging in this way allows us to understand how, from Hegel's point of view, the practical judgments found in Aristotle's *Nicomachean Ethics* provide the appropriate content for a type of inference that extends from a *mere fact* to *a cause* of that fact. For *his* part, Peirce would articulate his approach to logic within an overall pragmatist or "pragmaticist", framework, the maxim for which states that "the entire meaning and significance of any conception lies in its *conceivably* practical bearings".[69] From this perspective, Peirce, like Hegel, would make judgments of the *goodness* or *badness* of acts of judging and inferring—one's own acts or those of others—central to the very discipline of logic itself. It is this that binds the normativity of logic to that of ethics and, ultimately, "esthetics".

As a model for both, I have suggested the diagrammatic form of reasoning invoked by Aristotle in relation to his account of practical deliberation in *Nicomachean Ethics.* Most directly, diagrams give concrete representation to the notions of order, direction and reversal of direction, invoked by both Hegel and Peirce, but in Peirce in a self-conscious way, in their attempts to clarify the relations among different inferential forms. In the case of Hegel in particular, disambiguating different types of non-deductive inference implicit in Aristotle's notion of *epagoge* has been presented as linked to his disambiguation of the categories of singularity and particularity. In turn, I have suggested that Aristotle's blurring of these distinctions may have been linked to his adoption of the apparent elimination of numbers in Eudoxus' approach to geometry. All of these considerations, I suggest, conform to the largely Platonic approach to mathematics found in Hegel. Logic for Hegel is, of course, not mathematics, but the mathematical issues covered in Book I of *The Science of Logic*, unsurprisingly, reappear as *aufgehoben* in his discussion of Aristotle's logic in Book III.

[69] PEIRCE. **The Essential Peirce, volume 2,** p. 145.

In the Prologue to his *Commentary on Euclid's Elements*, Proclus repeats Plato's assignment of the objects of mathematics in the *Republic* to a "middle ground" between the ideal and material realms.

> To indivisible realities he assigned intellect … to divisible things in the lowest level of nature, that is, to all objects of sense-perception, he assigned opinion … whereas to intermediates, such as the forms studied by mathematics, which fall short of indivisible but are superior to divisible nature, he assigned understanding (PROCLUS, 1970, p. 40).

Nevertheless, there is a partition within this intermediate mathematical realm in that geometry "occupies a place second to arithmetic, which completes and defines it".[70] As Nicomachus of Gerasa had earlier pointed out,

> if geometry exists, arithmetic must also needs be implied, for it is with the help of this latter that we can speak of triangle, quadrilateral, octahedron, icosahedron …. But on the contrary, 3, 4, and the rest might be without the figures existing to which they are given names (NICOMACHUS OF GERASA, 1926).[71]

In some way, for these neo-Platonists, geometry has greater connection to the material realm, for although the objects from which Euclidean geometry starts, "the point without parts, the line without breadth, the surface without thickness", and so on, are not themselves observable entities,

> if the objects of geometry are outside matter, its ideas pure and separate from sense objects, then none of them will have any parts or body or magnitudes. For ideas can have magnitude, bulk, and extension in general only through the matter which is their receptacle (PROCLUS, 1970, p. 40).

To express this in another way, geometry, especially when its *applied* "problematic" or "analytic" dimension is considered, is a discipline fashioned for practical employment in the material world.

Despite not being *mathematical*, like Peirce's logic, Hegel's logic might nevertheless be illuminated by his idea *about* mathematics. It was only around the early part of the nineteenth

[70] PROCLUS. **A Commentary,** p. 40.
[71] NICOMACHUS OF GERASA. **Introduction to Arithmetic.** Trans. M.L. D'Ooge, New York: Macmillan Co., 1926. This work was in Hegel's personal library. C.f., MENSE, A. Hegel's Library: The Works on Mathematics, Mechanics, Optics and Chemistry. In: Perry, M J. (ed.), **Hegel and Newtonianism**. Dordrecht: Kluwer, 1993, pp. 672-673.

century that mathematics was being divided into "pure" and "applied" aspects and Hegel clearly thought of it as essentially *applied.* Thus, in his 1801 *Dissertation*, he would hold that

> the whole of mathematics must not be regarded as purely ideal or formal, but also as real and physical (HEGEL, 2002, p. 175).[72]

Within mathematics he emphasized geometry as linked to the material world, effectively considering geometry as the science of space itself. Like Proclus and Nicomachus, he stressed the dependence of geometry on arithmetic.

> Spatial magnitude has only delimitation in general; when considered as an absolutely determined quantum, it requires number. Geometry as such does not *measure* spatial figures – is not an art of measuring – but only *compares* them (HEGEL, 2010, p. 170; 21.196).

But this does not simply mean that arithmetic in a unilateral manner "completes and defines" geometry as it had for the neo-Platonists held. Arithmetic is as equally dependent on relations among continuous magnitudes as geometry is on discrete magnitudes.

> What is overlooked in the ordinary representations of continuous and discrete magnitude is that *each* of these magnitudes has both moments in it, continuity as well as discreteness, and that the distinction between them depends solely on which of the two is the *posited* determinateness and which is only implicit (HEGEL, 2010, p. 166; 21.190).

With this idea of reciprocal dependence of real and ideal elements, Hegel could stress those "qualitative" or "geometrical" aspects of thought itself that appeal to spatial- or temporal-like relations of continuity, order and orientation which enable us, as necessarily embodied and located beings in the world, to reason about it *from* somewhere within it. The interweaving of arithmetic and geometry within our mathematical practices and the interweaving of similarly arithmetical and geometrical aspects of *logic* within our thinking means that logic cannot be a purported science of some one-sided realm of ideal and thinkable entities that can be considered in abstraction from the concrete world towards which thought is otherwise directed. With this, the logic of Hegel, like that of Peirce, would break with the standard "formal" approach to logic that has persisted from the work of Aristotle, as conventionally understood, through to logic's

[72] HEGEL, G.W.F. On the Orbits of the Planets. **Miscellaneous Writings of G. W. F. Hegel**, J. Stewart (ed.). Evanston: Northwestern University Press, 2002.

modern "symbolic" revolutions—the types of logic Hegel would relentlessly criticise as the logic of the "mere understanding".

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